The prediction of vibrations for light structures in presence of moving people

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ABSTRACT

Recently, a model to describe the vibration of light structures (e.g. footbridges, staircases) was proposed by the authors of this paper. Such a model was developed with the aim of being accurate with a high number of people occupying the structure for long times. The present paper analyses the behaviour of the same model in the case of transient excitation of the structure. This allows to assess the accuracy of the model also in this further situation.

KEYWORDS: Human structure interaction; ground reaction force; dynamics; vibration; slender structure

1 Introduction

The dynamics of structures occupied by pedestrians is a widely studied topic. Many works treated the vibration serviceability issues of civil structures, with detailed focus on Human-Structure Interaction (HSI) and human induced vibrations [1][2][3][4][5]. Footbridges [6][7][8][9][10][11][12], grandstands of stadia [13][14][15], staircases [16][17][18], and other pedestrian structures [19][20][21] have been extensively investigated. Moreover, international standards and codes [22][23][24][25][26] have been developed with the aim of both designing and evaluating the structure dynamics under the crowd action and they are the usual reference when vibration serviceability is assessed. Unlike other standards, a recent guidance [27] (Joint Working Group, 2008) regarding dynamic performance requirements for permanent grandstands subjected to crowd action recommends to consider HSI.

There are many works which proposed models to describe HSI and/or predict the structure dynamics under the action of pedestrians (e.g., [7][19][20]). This work treats a model already presented in the literature [17][18], which describes the action of each single person on the structure and considers the structure as a multi-degrees-of-freedom system. Therefore, even structures with coupled modes can be considered when computing the structural response due to people walking. The focus of the present paper is the analysis of the accuracy of this model in case of few people on the structure for a short time. Indeed, the model was originally developed for the case of many pedestrians on the structure for a long time.

The paper is structured as follows: Section 2 describes the theoretical model used to predict structural vibrations in presence of people; Section 3 describes the tests carried out for the aim of the paper; finally, Section 4 discusses the results.

2 The model

When a pedestrian is in contact with a point of a structure, he/she produces a Ground Reaction Force (GRF). The GRF is the total force exchanged between the person and the structure. It is possible to see the GRF as the sum of a passive GRF (PGRF) and an active GRF (AGRF) [17][18]. The PGRF is the force generated by structural movement. Indeed, when the structure vibrates, it excites the person. If the person is considered as a dynamic system, he/she starts to vibrate as well and thus exerts a force on the structure. This force is named PGRF. Conversely, the AGRF is generated by the person's active movement. The AGRFs do not depend on (and are not generated by) the vibration of the structure behind the person and are caused by the active movement of the person. The AGRF can be described as the force exerted by a moving person on a structure with an infinite stiffness.

Therefore, the PGRF depends on the dynamic features of the person and on the motion of the structure. On the other hand, the AGRF is caused by an active movement of the pedestrian and is not related to the dynamics of the structure.

According to Figure 1, the dynamics of the structure occupied by people can be described as:

$$\mathbf{x}(\omega) = \mathbf{G}(\omega)(-\mathbf{f}^{\mathrm{ACTIVE}}(\omega) - \mathbf{f}^{\mathrm{GR}}(\omega) + \mathbf{f}(\omega)) \qquad [1]$$

Where **G** is the matrix containing the frequency response functions (FRF) of the empty structure, **x** is the vector of the displacements of the degrees-of-freedom in which the structure has been discretised, **f** is the vector of the external forces, $\mathbf{f}^{\mathrm{GR}}$ is the vector of the PGRFs, and $\mathbf{f}^{\mathrm{ACTIVE}}$ is the vector of the AGRFs. The expression of **G** is [28][29]:

$$\mathbf{G}(\omega) = \sum_{j=1}^{n} \frac{\boldsymbol{\phi}_j \boldsymbol{\phi}_j^{\mathrm{T}}}{\omega_j^2 - \omega^2 + 2\mathrm{i}\zeta_j \omega \omega_{\mathrm{j}}} \qquad [2]$$

where ω_{j} is the jth eigenfrequency, ζ_j is the associated non-dimensional damping ratio and $\boldsymbol{\phi}_j$ is the jth mode shape vector (scaled to the unit modal mass) evaluated/measured at discrete points. The superscript T indicates transposition. Finally, *n* is the (arbitrary) number of considered modes, i is the imaginary unit, and ω is the circular frequency. Since the eigenvector components are known at discrete (n_d) points, the matrix **G**(ω) is an $n_d \times n_d$ matrix containing the FRFs for these degrees-of-freedom.

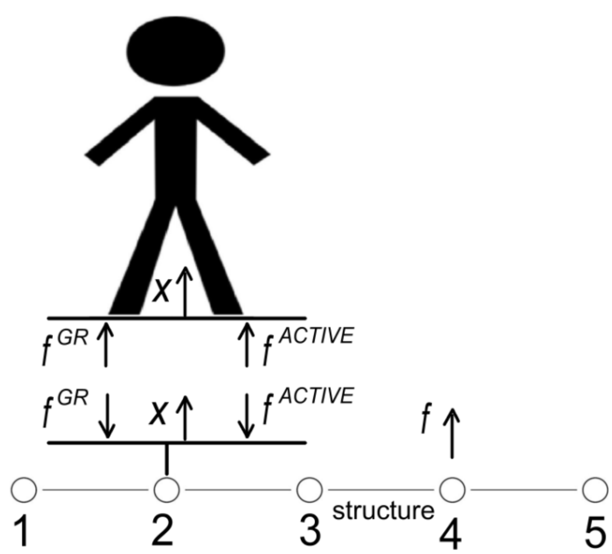

Figure 1: Human-structure interaction: AGRFs ($\mathbf{f}^{\mathrm{ACTIVE}}$) and PGRFs ($\mathbf{f}^{\mathrm{GR}}$)

Equation 1 can be used to estimate the structural response due to people movement on the structure. To this purpose, the AGRFs and PGRFs have to be estimated.

As for the AGRFs, it is possible to build a database of forces for the considered kind of movement (e.g., as in [17] for people ascending and descending staircases).

As for PGRFs, we use the apparent mass of the pedestrians, which is the frequency response function (FRF) between the acceleration at the contact point between the pedestrian and the structure and the consequent force exerted by the pedestrian on the structure. This FRF can be measured as described in [17][30][31][32].

However, we must consider that people change posture during motion and that they move over the structure so that the PGRFs change point of application in time. As for the former point, an equivalent apparent mass M^*_{eq} is defined considering many different postures of the subject during the motion. Therefore, each step is split in *P* postures and the apparent mass $M^*_{a,i}$ is estimated (e.g., with experimental tests as in [17]) for each of them. Actually, each $M^*_{a,i}$ can be calculated as the average of the apparent mass of different people in the given posture. Hence, M^*_{eq} is calculated as:

$$M^*_{eq}(\omega) = \sum_{\mathrm{i}=1}^{P} \alpha_i \, M^*_{a,i}(\omega) \qquad [6]$$

where the weighting coefficients α_i are set in order to describe properly the amount of time spent by the pedestrians in the corresponding posture within the cycle time *T* (i.e., the time of a single step). In other words, *P* postures are frozen within the movement cycle and each of them is treated as a static posture.

Thus, each person produces a PGRF related to the apparent mass M^*_{eq}. Then, the PGRFs can be applied to the structure with two different approaches:

1. each PGRF (i.e., one for each pedestrian on the structure) is seen as a moving excitement. Therefore, the whole system is time-variant;
2. a fraction of the apparent mass $m^*{}_{fr}(\omega)$ is calculated:

$$m^*_{fr}(\omega) = \frac{m}{n_d} M^*_{eq}(\omega) \qquad [7]$$

   Where n_d is the number of degrees-of-freedom in which the structure is discretised, and *m* is the number of pedestrians on the structure. Then, $m^*_{fr}(\omega)$ is applied to each of the n_d degrees-of-freedom. Therefore, the PGRF in each degree-of-freedom can be expressed as:

$$f_i^{\mathrm{GR}}(\omega) = m^*_{fr}(\omega)\ddot{x}_i(\omega) = -m^*_{fr}(\omega)\omega^2 x_i(\omega) \qquad [8]$$

   In terms of the full displacement vector $\mathbf{x}(\omega)$, the $\mathbf{f}^{\mathrm{GR}}$ can be described as [17][18]:

$$\mathbf{f}^{\mathrm{GR}}(\omega) = \mathbf{W_n}\mathbf{H}\mathbf{W}_\mathbf{n}^{\mathrm{T}}\mathbf{x}(\omega) = -\omega^2 m^*_{fr}(\omega)\mathbf{W_n}\mathbf{x}(\omega) \qquad [9]$$

   where $\mathbf{W_n}$ is a n_d x m matrix describing the connection of the *m* subjects with the structure degrees-of-freedom, $\mathbf{H}(\omega)$ is a m x m diagonal matrix containing the fractions of the equivalent apparent mass (i.e. $\mathbf{H}(\omega) = -\mathbf{W_n}\omega^2 m^*_{fr}(\omega)$). Substituting Equation 9 into Equation 1, we obtain (neglecting **f**):

$$[\mathbf{G}^{-1}(\omega) + \omega^2 m^*_{fr}(\omega)\mathbf{W_n}]\mathbf{x}(\omega) = \mathbf{G}_{\mathrm{H}}^{-1}(\omega)\mathbf{x}(\omega) = -\mathbf{f}^{\mathrm{ACTIVE}}(\omega) \qquad [10]$$

where $\mathbf{G_H}(\omega)$ is the n_d x n_d matrix representing the equivalent set of FRFs describing the dynamic behaviour of the joint system composed by the structure and the people. Clearly, the behaviour of this coupled system is an average behaviour because m^*_{fr} is employed.

The second approach of the previous list assumes a fixed form of $\mathbf{G}_{\mathrm{H}}(\omega)$ in time. Therefore, this assumption makes the simulation of the structure response fast and easy under the computational point of view.

The response of the structure to the movement of people can be finally calculated as the convolution between the AGRFs and the unit impulse response functions (IRF) of the coupled system. These IRFs can be found by applying the inverse Fourier transform to the FRFs composing $\mathbf{G}_{\mathrm{H}}(\omega)$. More details about this model can be found in [16][17][18].

When the number of people on the structure is high, and/or people occupy the structure for long times, the accuracy of this easy-to-apply approach is high [17]. Indeed, in this case the approximation due to the use of $m^*{}_{fr}(\omega)$ results to be acceptable and does not introduce accuracy worsening in result estimation.

The next sections show the performance of this model when applied to the case of few people on the structure for a limited amount of time.

3 Tests

This section describes the experimental campaign carried out to validate the model presented in Section 2. A staircase (made up of steel and marble, length 12.03 m, width 1.80 m, and height 5.22 m) was used as test-structure. The modal parameters of the empty structures were identified by means of experimental modal analysis [33] and the values of the eigenfrequencies and non-dimensional damping ratios are provided in Table 1. The modes taken into account are those in the frequency range 0-15 Hz, which is the frequency band where the pedestrians are mostly able to influence the structural dynamics.

Mode number	$\omega_j/(2\pi)$ [Hz]	ζ_j [%]
1	7.84	0.33
2	8.89	0.43

Table 1: Modal data identified for the test-structure in the frequency range 0-15 Hz (empty structure)

The tests carried out with pedestrians crossing the stair were several. Here, we discuss two of them, which are gathered in Table 2, because their results are representative also of the results of the other tests. Each test was repeated several times with different pedestrians (for all of them apparent mass curves and AGRF time-histories were stored thanks to dedicated experimental tests). The structural response was measured by means of accelerometers.

Test ID	Type of test	Number of pedestrians
A	Test with pedestrians continuously walking on the stair in loop	1
B	Test with pedestrians crossing the stair once	3 (2 descending the staircase and 1 ascending)

Table 2: Tests description

The same tests were also simulated using the model described previously. Each test was simulated 100 times, extracting randomly the pedestrians and the AGRFs from the database. This allowed to take into account the natural dispersion of experimental results. Since the results showed Gaussian distributions, the results of the model are described by the mean value of the results of the 100 simulations plus/minus twice the standard deviation of the results (i.e. with a confidence level of about 95% [34]).

The results are expressed in terms of root-mean-square (RMS) of one of the acceleration signals showing the highest structural responses among all the accelerometers used to monitor the structural response. The same degree-of-freedom was considered in the simulation results. Moreover, a moving RMS was calculated as well (the moving RMS was calculated every 3 s). Its maximum value was also taken into account to express experimental and numerical results. This maximum RMS will be named here MRMS.

The RMSs and MRMSs were calculated in the frequency range 0-12 Hz.

4 Results

Figure 2 and Figure 3 show the results for test A and test B, respectively. It is evident that when the staircase is occupied for long times (i.e. test A, Figure 2), the model is still able to predict the vibration levels, both in terms of RMS and MRMS. Indeed, the order of magnitude of experimental and numerical result is the same (there are just few experimental results below the expected RMSs). Conversely, when the time length of the test is short (i.e. test B, Figure 3), the numerical results clearly overestimate the experimental RMSs. As for the MRMSs, the model still overestimates the experiments because the interval provided by the model stretches over high MRMS values. However, the overestimation is not as much as in the case of the RMS.

Therefore, when long-time tests are taken into account, the model is able to correctly estimate RMSs and MRMSs. As for short tests, the model is still able to describe the vibration of the structure occupied by pedestrians. Indeed, the order of magnitude of RMSs and MRMSs values is correct and many experimental results fall into the intervals provided by the model. However, overestimations of both RMSs and MRMSs are evident.

This means that an accurate estimation of the vibration levels under operating conditions in case of transient excitation (i.e. situations comparable to the short test discussed here) would require the use of a more complex model able to properly describe the evolution of the PGRFs. This can be accomplished by developing time domain models where the PGRFs are described in time and space. Therefore, a model based on time integration is under development by the authors of the present paper. This model will be able to describe the position and the effect of each pedestrian on the structure as function of time. Such an approach is expected to produce results closer to the experimental evidence when transients are taken into account.

5 Conclusions

The paper has dealt with the problem of human-structure interaction. Particularly, the issue treated here has regarded a model to describe the vibration of light structures (e.g. staircases) proposed recently by the authors of this paper. This model was developed with the goal of being accurate in case of a high number of pedestrians on the structure for long times. The present paper has analysed the behaviour of the same model in the case of transients and few people on the structure. This has allowed to assess the accuracy of the model also in this further situation. The main outcome is the need of a more refined model for transient situations, which must be able to properly describe the contribution of the passive ground reaction forces in time and space.

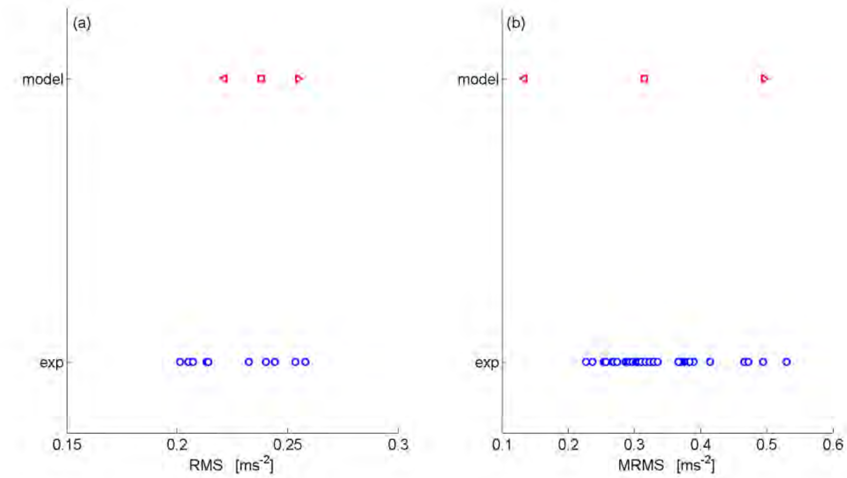

Figure 2: Experimental and numerical results for test A. Numerical results are expressed in terms of mean value (square) plus/minus twice the standard deviation (triangles).

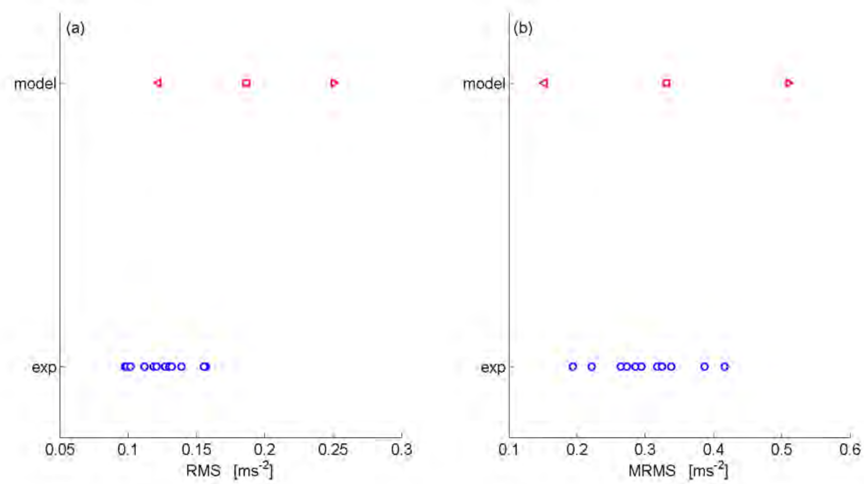

Figure 3: Experimental and numerical results for test B. Numerical results are expressed in terms of mean value (square) plus/minus twice the standard deviation (triangles).

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